Jamming and Fluctuations in Granular Drag

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We investigate the dynamic evolution of jamming in granular media through fluctuations in the granular drag force. The successive collapse and formation of jammed states give a stick-slip nature to the fluctuations which is independent of the contact surface between the grains and the dragged object, thus implying that the stress-induced collapse is nucleated in the bulk of the granular sample. We also find that while the fluctuations are periodic at small depths, they become "stepped" at large depths, a transition which we interpret as a consequence of the long-range nature of the force chains.

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Materials in granular form are composed of many solid particles that interact only through contact forces. In a granular pile, the strain resulting from the grains' weight combines with the randomness in their packing to constrain the motion of individual grains. This leads to a "jammed" state [1] which also characterizes a variety of other frustrated physical systems, such as dense colloidal suspensions and spin glasses [1,2]. Not surprisingly, the dynamics of granular materials, while in many ways analogous to those of fluids, are, in fact, quite different due to this frustration of local motion [3]. The effects of this jamming are also manifested in static properties, leading to inhomogeneous stress propagation through force chains of strained grains [4–10] and arch formation [11]. Although jamming in granular materials has previously been discussed in the context of the gravitational stress induced by the weight of the grains, it can result from any compressive stress. For example, a solid object being pulled slowly through a granular medium is resisted by local jamming, and can advance only with large scale reorganizations of the grains [12]. The granular drag force originates in the force needed to induce such reorganizations, and thus exhibits strong fluctuations which qualitatively distinguish it from the analogous drag in fluids [13].

In this Letter we investigate the dynamic evolution of jamming in granular media through these fluctuations in the granular drag force. The successive collapse and formation of jammed states give a stick-slip character to the force with a power spectrum proportional to $1/f^2$. We find that the stick-slip process does not depend on the contact surface between the grains and the dragged object, and thus the slip events must be nucleated in the bulk of the grains opposing its motion. While the fluctuations are remarkably periodic for small depths, they undergo a transition to "stepped" motion at large depths. These results point to the importance of the long-range nature of the force chains to both the dynamics of granular media and the strength of the granular jammed state.

The experimental apparatus, shown in Fig. 1(a), consists of a vertical steel cylinder of diameter d_c inserted to a depth H in a bed of glass spheres [14] moving with constant speed [13]. The cylinder is attached to a fixed force cell [15], which measures the force $F(t)$ acting on the cylinder as a function of time. The bearings on the cylinder's support structure allow the cylinder to advance freely only in the direction of motion so that the force cell alone is opposing the drag force from the grains. We incorporate a spring of known spring constant, k, between the cylinder and the force cell—choosing k (between 5 to 100 N/cm), so that this spring dominates the elastic response of the cylinder and all other parts of the apparatus. We vary the speed (v) from 0.04 to 1.4 mm/s, the depth of insertion (H) from 20 to 190 mm, and the cylinder diameter (d_c) from 8 to 24 mm, studying grains of diameter (d_g) 0.3, 0.5, 0.7, 0.9, and 1.1 mm [16]. The force is recorded

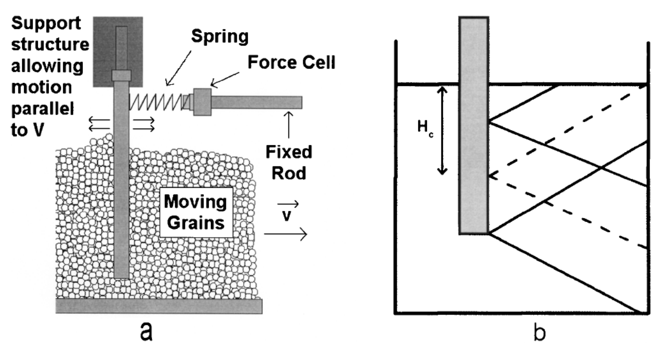

FIG. 1. (a) An enlarged view of the apparatus used for measuring the drag force. The grains were contained within a 25 cm diameter rotating bucket as described in detail previously [13]. (b) Schematic illustration of the force chains originating at the surface of an object dragged to the right in a finite container, where H_c corresponds to the depth at which the force chains all terminate at the walls of the container. Of course, this picture is highly simplified since the actual force chains bifurcate and follow nonlinear paths.

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at 150 Hz, and the response time of the force cell and the amplifier are <0.2 ms. Note that these experiments are conducted in dense static granular media as opposed to drag in dilute or fluidized media which have been studied both theoretically [17] and experimentally [18].

Consistent with earlier results [13], we find that the average drag force on the cylinder is independent of v and k, and is given by $\overline{F} = \eta \rho g d_c H^2$, where η characterizes the grain properties (surface friction, packing fraction, etc.), ρ is the density of the glass beads, and g is gravitational acceleration. As shown in Fig. 2, however, $F(t)$ is not constant, but has large stick-slip fluctuations consisting of linear rises associated with a compression of the spring and sharp drops associated with the collapse of the jammed grains opposing the motion. The linear rises in $F(t)$ correspond to the development of an increasingly compressed jammed state of the grains opposing the motion. We find that, independent of the depth, the slopes of the rises are given by $\frac{1}{v} dF/dt = k$ for all springs with $k < 100$ N/cm, confirming that the spring dominates the elasticity of the apparatus. This result also implies that, during the rises, the jammed grains opposing the cylinder's motion do not move relative to each other or the cylinder. The power spectra, $P(f)$ [the squared amplitudes of the Fourier components of $F(t)$], are independent of both the elasticity of the apparatus and the rate of motion, so that the scaled spectra $kvP(f)$ vs f/kv collapse in the low frequency regime ($f < 10$ Hz) [5]. This indicates that the fluctuations reflect intrinsic properties of the development and collapse of the jammed state rather than details of the measurement process. The power spectra also exhibit a distinct power law, $P(f) \propto f^{-2}$, over as much as two decades in frequency (Fig. 3), a phenomenon which has been reported in other stick-slip processes and is intrinsic to random sawtooth signals [19].

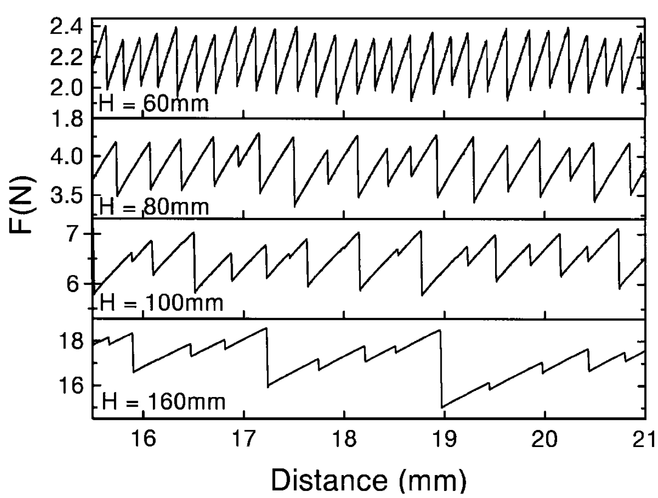

FIG. 2. The characteristic fluctuations in the drag force at four different values of H for $d_c = 10$ mm. Note the transition from purely periodic fluctuations $H \leq 60$ to stepped fluctuations with increasing depth $H \geq 100$.

During each fluctuation the force first rises to a local maximum value (F_{max}), and then drops sharply (by an amount ΔF), corresponding to a collapse of the jammed state. The force from the cylinder propagates through the medium via chains of highly strained grains, and a collapse occurs when the local interparticle forces somewhere along one of the chains exceed a local threshold. The corresponding grains then slip relative to each other, which in turn nucleates an avalanche of grain reorganization to relieve the strain. This allows the cylinder to advance relative to the granular reference frame, with a corresponding decompression of the spring and a drop in the measured force $F(t)$ [20]. The interparticle forces within the force chains are largest at the cylinder's surface, where the chains originate, and their magnitude decreases as we move away from the cylinder, and the force chains bifurcate. Consequently, one might expect that the reorganization is nucleated among grains in contact with the cylinder, but we find no significant change either in $\overline{F}$ or in the fluctuations when we vary the coefficient of friction between the grains and the cylinder by a factor of 2.5 (substituting a teflon-coated cylinder for the usual steel cylinder). As demonstrated in the inset of Fig. 3, the power spectra are also almost unchanged by substituting a half cylinder (i.e., a cylinder bisected along a plane through its axis and oriented so that the plane is normal to the grain flow) for a full cylinder of the same size, indicating that the geometric factors do not play a significant role either. These results indicate that the fluctuations are not determined by the interface between the dragged object and the medium, but rather that the failure of the

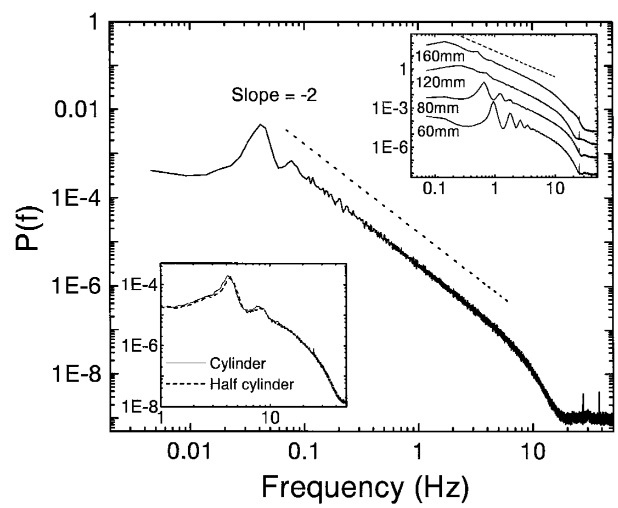

FIG. 3. The power spectrum of the fluctuations in the drag force taken for $d_g = 1.1$ and $d_c = 16$ mm at depth $H = 60$ mm with $v = 0.05$ mm/s and $k = 5$ N/cm to increase the dynamic range. Note that in both the periodic and the stepped regimes the spectrum has a long f^{-2} regime as shown in the upper inset (shifted vertically for clarity) ($d_c = 16$ mm). The lower inset ($d_g = 1.1$, $d_c = 19$ mm) shows that the power spectrum does not change when a half cylinder is substituted for a full cylinder.

jammed state is nucleated *within the bulk of the medium.* In this respect, the fluctuations are rather different from either ordinary frictional stick-slip processes which originate at a planar interface between moving objects, or the motion of a frictional plate on top of a granular medium [21].

A striking feature of the data is that the fluctuations change character with depth. For $H < H_c \approx 80$ mm the fluctuations are quite periodic, i.e., $F(t)$ increases continuously to a nearly constant value of $F_{\max}$ and then collapses with a nearly constant drop of ΔF (Fig. 2). As the depth increases, however, we observe a change in $F(t)$ to a stepped signal: Instead of a long linear increase followed by a roughly equal sudden drop, $F(t)$ rises in small linear increments to increasing values of $F_{\max}$, followed by small drops (in which ΔF is on average smaller than the rises), until $F_{\max}$ reaches a characteristic high value, at which point a large drop is observed. This transition from a periodic to a stepped regime is best quantified in Fig. 4, where we plot the depth dependence of $\overline{\Delta F}$ and the relative standard deviation of ΔF, $\sigma_n = \sigma_{\Delta F}/\overline{\Delta F}$. In the periodic regime, $\overline{\Delta F}$ rises due to the increase in $\overline{F}$. As the large uniform rises of the periodic regime are broken up by the small intermediate drops, however, $\overline{\Delta F}$ shows a local minimum and $\sigma_{\Delta F}/\overline{\Delta F}$ increases drastically, saturating for large depths. The transition is also observed in the power spectra as shown in the upper inset of Fig. 4. For low depths the power spectra display a distinct peak characteristic of periodic fluctuations, but these peaks are suppressed for $H > H_c$ in correlation with the changes in $\sigma_{\Delta F}/\overline{\Delta F}$ and the qualitative character of $F(t)$.

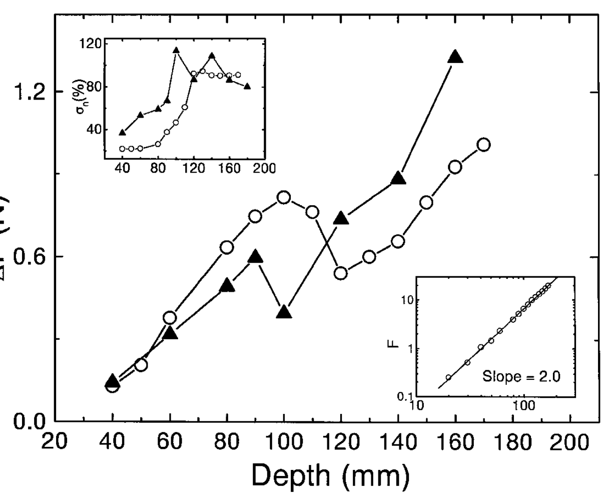

FIG. 4. The transition from periodic to stepped fluctuations as shown through the magnitude of the average drop $\overline{\Delta F}$, for two different container sizes: circles—large container (D = 25 mm); triangles—small container (D = 10 mm). The upper inset shows the relative magnitude of the standard deviation $\sigma_n = \sigma_{\Delta F}/\overline{\Delta F}$. The transition occurs at smaller H_c in the smaller container (d_c = 10 mm). The lower inset shows the depth dependence of the average drag force for d_g = 1.1 and d_c = 10 mm; there is no change in the slope at H_c. The solid line has a slope of 2.

The transition from a periodic to a stepped signal is rather unexpected, since it implies qualitative changes in the failure and reorganization process as H increases and the existence of a critical depth, H_c. An explanation for H_c could be provided by Janssen's law [22] which states that the average pressure (which correlates directly with the local failure process) should become depth independent below some critical depth in containers with finite width. This should not occur in our container, however, which has a diameter of 25 cm, much larger than H_c. Furthermore we see no deviation in the behavior of $\overline{F}(H)$ from $\overline{F} \propto H^2$, which depends on the pressure increasing linearly with the depth (Fig. 4, lower inset).

In order to account for the observed transition, we must inspect how the force chains originating at the surface of the cylinder nucleate the reorganizations. The motion of the cylinder attempting to advance is opposed by force chains that start at the cylinder's surface and propagate on average in the direction of the cylinder's motion. These force chains will terminate rather differently depending on the depth at which they originate, as shown schematically in Fig. 1(b). For small H, some force chains will terminate on the top surface of the granular sample, and the stress can be relieved by a slight rise of the surface. Force chains originating at large depths, however, will all terminate at the container's walls. Since the wall does not permit stress relaxation, the grains in these force chains will be more rigidly held in place. According to this picture, H_c corresponds to the smallest depth for which all force chains terminate on the wall. When the cylinder applies stress on the medium, the force chains originating at small H ($H < H_c$) reduce their strain through a microscopic upward relaxation of chains ending on the free surface. By contrast, the higher rigidity of force chains originating at $H > H_c$ impedes such microscopic relaxations. Thus a higher proportion of the total force applied by the cylinder will be supported by those force chains, enhancing the probability of a local slip event occurring at high depths. Such a slip event would not necessarily reorganize the grains at all depths (for example, the grains closer to the surface may not be near the threshold of reorganization), thus the slip event might induce only a local reorganization and a small drop in $F(t)$. The large drops in $F(t)$ would occur when force chains at all depths are strained to the point where the local forces are close to the threshold for a slip event. This scenario also explains why $\overline{F}(H)$ does not change at H_c, since $\overline{F}$ is determined by the collective collapse of the jammed structure of the system.

According to this picture, the transition is expected at smaller depths in smaller containers since the force chains would terminate on the walls sooner [see Fig. 1(b)]. Indeed, as we show in Fig. 4, the transition does occur at a depth approximately 20 mm smaller when the measurements are performed in a container 2.5 times smaller (with a diameter of 100 mm). Furthermore, we fail to observe the periodic fluctuations in any grains with diameters

1.4 mm or larger [23], which is consistent with the suggested mechanism, since larger grains correspond to a smaller effective system size.

It is interesting to compare our results with those of Miller *et al.* [5] who studied fluctuations in the normal stress at the bottom of a sheared granular annulus. Those fluctuations were also independent of the rate of motion and demonstrated the long-range nature of the vertical force chains. In the present experiments, we confirm that force chains originating from a horizontal stress are also long range through our observation of the transition at depth H_c. Since for small grains ($d_g \leq 1.1$ mm) H_c has no measurable dependence on grain size, we find in agreement with Miller *et al.* that the nature of the force chains is not strongly dependent on grain size. Our observations also shed light on the implications of the long-range force chains for granular dynamics and the nature of jamming in granular materials. The crossover at H_c suggests that drag fluctuations in an infinitely wide container would be periodic, but that the finite size of a real container destroys the periodicity. In other words, the finite size of a container relative to these chains reduces the strength of jammed granular states within the container. These results point to the need for a better understanding of the detailed dynamics of force chains—both how they form when stress is applied to a granular medium and how they disperse geometrically from a point source of stress—in order to gain an understanding of slow granular flows.

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